M.V. Lomonosov Moscow State University

Moscow School of Economics

Syllabus

Modern Mathematical Tools of Economic Analysis

First semester 2013/2014

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The goal of this course is to give an introduction to some branches of mathematics which are extensively used in the modern economic theory, particularly in macro- and microeconomics. Studying this curse, students have to learn how such tools as convex programming, maximum principle and dynamic programming as well as fixed point theorems may be used to construct and investigate economic models.

The course is designed for first-year students of Master of Science Program in Economic Theory

Prerequisites: Calculus, Linear algebra, Microeconomics 1,2, Macroeconomics 1,2.

The course includes 15 lectures and 11 seminars.

The final grade of the course is computed as weighted mean in which the grade for seminars and home assignments has weight 0.2 and the grade for exams has weight 0.8.

Course outline

I. Convex Programming

Introduction

Mathematics in Economics Theory: The history and patterns

References: [1].

Topic 1. Elements of convex analysis.

Convex sets. Convex polyhedrons. Convexity of technological and consumption sets.

Properties of convex sets. Intersection, geometric sum and direct product of convex sets. Dimension of convex set. Extreme points of convex sets. Separation theorems.

Convex (concave) and quasi-convex (quasi-concave) functions. Conditions of convexity (concavity) in differential form. Utility functions and production functions, the meaning of the assumptions of convexity and quasi- convexity.

Properties of convex and quasi-concave functions. Continuity and differentiability of convex functions.

References: [1], [4], [9], [10], [18], [13]-[15]

Topic 2. Elements of convex programming.

Extremal problems. Local and global optima.

Unconstrained maximization; necessary and sufficient optimality conditions.

Convex programming (CP), Lagrange function and its saddle points.

Kuhn-Tucker theorem for CP problems. Interpretation of Lagrange multipliers. Slater’s condition. Necessary and sufficient optimality conditions for CP problem in the differential form.

Envelope theorem for CP problems and its economic interpretation.

Economic applications: supply and demand functions; Ramsey's model in discrete time.

References: [3], [4], [9], [10], [12], [18]

II. Maximum principle and dynamic programming

***Topic* 3.** Differential equations and stability theory

Ordinary differential equations. Existence and uniqueness of solutions. Linear differential equations with constant coefficients. Continuity and differentiability of solutions on parameters.

Stability theory. Lyapunov and asymptotic stability. First Lyapunov's method.

Second Lyapunov's method. Stability of price regulation processes.

References: [16], [17]

***Topic* 4.** Maximum principle

Problems of optimal control in continuous time. Models of optimal economic growth, Ramsey's model on infinite and finite interval.

Maximum principle as a necessary condition of optimality. Interpretation of dual variables. Sufficiency of the maximum principle for convex problems.

Classic problem of calculus of variations in continuous time (CPCV). Euler equation and maximum principle.

The relationship between maximum principle and Kuhn-Tucker theorem in the case of discrete time.

References: [3], [5], [6], [8], [19]-[21].

***Topic* 5.** Dynamic programming

Dynamic programming method for optimal control problem in discrete and continuous time. Bellman's principle of optimality.

Bellman equation. Synthesis of optimal control.

The relationship between dynamic programming and maximum principle.

References: [6], [7], [18], [13], [19]-[21].

III. Equilibrium and Pareto optimality

***Topic* 6. Fixed point theorems**

Contracting - mapping principle and its applications.

Brouwer's and Kakutani's theorems. Applications: existence of Nash equilibrium, existence of competitive equilibrium.

References: [9] – [13], [16], [18]

Topic 7. Pareto optimality

Strong and weak Pareto optimality. Theorem on criteria convolution. Necessary and sufficient conditions of Pareto optimality. Pareto optimality and equilibrium states.

References: [9] – [13], [16], [18]

Seminars

1. Solutions of problems on the topics: convex functions and convex sets.

2. Calculation and investigation of supply and demand functions. Substitutability and complementarity of goods. Finding of equilibria for the simplest versions of Arrow-Debreu models. Comparative statics.

3-4. Differential equations, stability of solutions of differential equations.

5-7. Maximum principle in continuous time, Euler equation, Ramsey's model. Derivation of Euler equation from maximum principle Transversality conditions. Steady states.

7. Pareto optimality and equilibrium. Application of convolution theorem for determination of all Pareto-optimal states. First and second Welfare theorems. Efficiency of trajectories of economic growth.

8. Investigation of oligopolistic behavior. Solutions of monopoly models. Cournot and Bertrand Oligopolies.

References

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[18] S. Karlin Mathematical methods in game theory, programming and economics. Mir, Moscow, 1964.

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