Bitcoin and Cryptocurrencies Financial Modelling

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Overview of the Presentation

- (Very brief) introduction to cripto-currencies and bitcoin
- What is bitcoin's fundamental value? A review of financial and economic approaches
- Modelling bitcoin price dynamics
- Detecting bubbles and explosive behaviour in bitcoin prices
- Market price discovery
- Conclusions

(Very brief) introduction to Cripto-currencies and Bitcoin

The Bitcoin network uses cryptography to validate transactions during the payment processing and create transaction blocks. In particular, Bitcoin relies on two cryptographic schemes:

1. *digital signatures* (to exchange of payment instructions between the involved parties)
2. a *cryptographic hash function*: to maintain the discipline when recording transactions to the public ledger (known as *blockchain*)

(Very brief) introduction to Cripto-currencies and Bitcoin

Digital signatures are used to authenticate digital messages between a sender and a recipient, and they provide:

1. *Authentication*: receiver can verify that the message came from sender
2. *Non-repudiation*: the sender cannot denyhaving sentthe message;
3. *Integrity*: the message was not altered in transit.

The use of digital signatures includes public key cryptography, where a pair of keys (open and private) are generated with certain desirable properties.

A digital signature is used for signing messages: the transaction is signed using a private key, and transferred to the Bitcoin network.

The members of the network can verify that the transaction came from the owner of the public key by taking the message, the signature, the public key and running a test algorithm

(Very brief) introduction to Cripto-currencies and Bitcoin

A *cryptographic hash function* takes as input a string of arbitrary length (the message m), and returns the string with predetermined length (the hash h).

The function is deterministic, which means that the same input m will always give the same output h. In addition,the function must also have the following properties:

1. *Pre-image resistance:*for a given hash h, it is difficult to find a message m such that hash$(m)=h$.
2. *Collision resistance*: for a given message m_1 it is hard to find another message m_2, such that hash $(m_1) =$ hash(m_2). In other words, a change in the message leads to a change in the hash.

The output of the hash function looks like to be random, although it is completely deterministic. The Bitcoin network mainly uses the secure hash algorithm SHA-256

(Very brief) introduction to Cripto-currencies and Bitcoin

From a technical standpoint, bitcoins stay in the Bitcoin network on bitcoin-addresses.

The ownership of a certain number of bitcoins is represented by the ability to send payments via the Bitcoin network using the bitcoins attached to these addresses.

In particular, every bitcoin address is indexed by a unique public ID, which is an alphanumeric identifier, which corresponds to the public key.

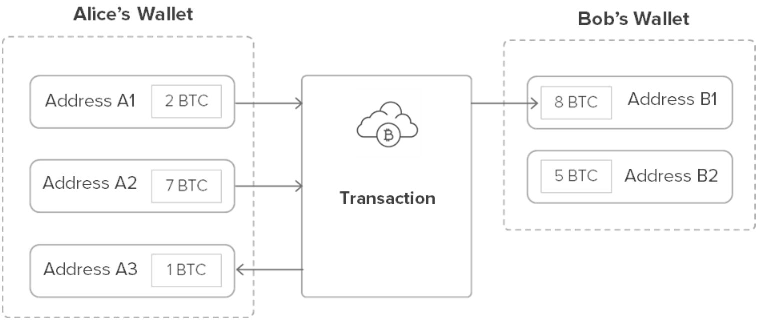
The private key controls the bitcoins stored at that address. Any payment (i.e. a *message*) which involved this address as the sending address must be signed by the corresponding private key to be valid.

$\Rightarrow$ In straight terms,the possession of bitcoins at a specified bitcoin address is given by the knowledge of the private key corresponding to that address.

(Very brief) introduction to Cripto-currencies and Bitcoin

The agents who process transactions in the Bitcoin network use a set of bitcoin addresses called the *wallet*, which is the set of bitcoin addresses that belong to a single person/entity.

Each transaction record includes one or more sending addresses (inputs) and one or more receiving addresses (outputs), as well as the information about how much each of these addresses sent and received:



(Very brief) introduction to Cripto-currencies and Bitcoin

After the initial check of the transaction signed messages, validation nodes in the Bitcoin network begin to compete for the opportunity to record a transaction in the blockchain.

1. Competing nodes start putting together transactions in a new block, which were executed since the last record in the blockchain.
2. The block is used to define a complex computing task based on the hash function. The node that first solves this task records the transactions on the blockchain and collects a reward.
3. The implementation of this scheme is the so-called *Hashcash* - a proof that the system is operating properly (proof-of-work), and whose aim is to ensure that the computers use a certain amount of computing power to perform a task (see Beck (2002) for more details).

(Very brief) introduction to Cripto-currencies and Bitcoin

The nodes that perform the process of the proof-of-work in the Bitcoin network are called *miners*.

These miners use their computing resources in this process with the goal to obtain the reward offered by the Bitcoin Protocol.

Usually the reward is a predetermined number of newly created bitcoins.

The rest of the reward (which is currently smaller), is a voluntary transaction fee paid by those executing the transaction to the miners for transaction processing.

What is bitcoin fundamental value? A review of financial and economic approaches

An upper bound: *Market Sizing*

Market sizing is basically the process of estimating the potential of a market and this is widely used by companies which intend to launch a new product or service.

Woo et al. (2013) in a Bank of America Merrill Lynch report estimated separately the value of bitcoin as a A) *medium of exchange* and as B) *store of value* and then summed them up to get a rough estimate of bitcoin fair value.

What is bitcoin fundamental value? A review of financial and economic approaches

1. More specifically, to compute the value as medium of exchange, they considered two uses for bitcoin: *e-commerce* and *money transfer*:

$$V_{e-commerce_t} = \frac{1}{10}\left(\sum_{i=1}^{10}\frac{HD_{US_{t-i}}}{C_{US_{t-i}}}\right)\cdot \text{B2C}_{\text{t-1}} \cdot Bitcoin_{share} \cdot \frac{GDP_{world_{t-1}}}{GDP_{US_{t-1}}}$$

which is approximately \$5bn worth of Bitcoins for the total global on-line shopping.

$$V_{money\ transfer_t} = \frac{1}{3}\left(MK_{WU_t} + MK_{MG_t} + MK_{E_t}\right)$$

that is, the the average market capitalization of Western Union, MoneyGram and Euronet (approximately \$ 4.5bn)

What is bitcoin fundamental value? A review of financial and economic approaches

2. Woo et al. (2013) suggested that the closest assets to bitcoin as a *store value* are probably precious metals or cash.

They suggested that the Bitcoin market capitalization for its role as a store of value could reach $5bn.

Interestingly, they noted that this value is close to the value of the total US silver eagles minted since 1986 (around $8bn - 12k tons)

$$V_{store\ of\ value_t} = 0.6 \cdot TSM_t \cdot P_{silver,t}$$

where TSM_t is the total sum of all US silver eagles minted since 1986 at time t, while $P_{silver,t}$ is the price for 1 troy ounce of silver at time t.

What is bitcoin fundamental value? A review of financial and economic approaches

Finally, Woo et al. (2013) computed the potential bitcoin fair value as

$$P_{bitcoin_t} = \frac{(V_{e-commerce_t} + V_{money\ transfer_t} + V_{store\ of\ value_t})}{TB_t}$$

where TB_t is the total number of bitcoin in circulation, thus obtaining a maximum fair value of Bitcoin approximately equal to 1300$ (**Woo et al. (2013) used data up to 2012**).

Finally, a similar approach is investigated by Huhtinen (2014), who considered the current money aggregates M2 for USD, EUR and JPY, and alternative scenarios for the portion of money supply that could be replaced by bitcoin, instead. He argues the most realistic replacement level is 0.1{%} and it could be achieved with a bitcoin valuation of 1573 euro.

A lower bound: *the marginal cost of bitcoin production*

Garcia et al. (2014) were the first to suggest that the fundamental value of one bitcoin should be at least equal to the cost of the energy involved in its production through mining.

$\Rightarrow$ lower bound estimate of bitcoin fundamental value.

More recently, a more refined model for the cost of bitcoin production was developed by Hayes (2015a,b). Variables to consider:

1. the cost of electricity, measured in cents per kilowatt-hour;
2. the energy consumption per unit of mining effort, measured in watts per GH/s (1 W/GH/s=1 Joule/GH);
3. the bitcoin market price;
4. the difficulty of the bitcoin algorithm;
5. the block reward (currently 12,5 BTC), which halves approx. every 4 years

A lower bound: *the marginal cost of bitcoin production*

In a competitive commodity market, an agent would undertake mining if the marginal cost per day (electricity consumption) were less than or equal to the marginal product (the number of bitcoins found per day on average multiplied by the dollar price of bitcoin).

Hayes (2015a,b) develops his model by assuming that a miner's daily production of bitcoin depends on its own rate of return, measured in expected bitcoins per day per unit of mining power.

The expected number of bitcoins expected to be produced per day can be calculated as follows:

$$BTC/day^* = [(\beta \cdot \rho)/(\delta \cdot 2^{32})] \cdot sec_{hr} \cdot hr_{day} \tag{1}$$

where β is the block reward (currently 12,5 BTC/block), ρ is the hashing power employed by a miner, and δ is the difficulty (which is expressed in units of GH/block).

A lower bound: *the marginal cost of bitcoin production*

The constant sec_{hr} is the number of seconds in an hour (3600), while hr_{day} is the number of hours in a day (24).

The constant 2^{32} relates to the normalized probability of a single hash per second solving a block, and is a feature of the 256-bit encryption at the core of the SHA-256 algorithm.

These constants which normalize the dimensional space for daily time and for the mining algorithm can be summarized by the variable θ, given by $\theta = 24hr_{day} \cdot 3600/2^{32}sec_{hr} =$ 0.000020116567616943. Equation (1) can thus be rewritten compactly as follows:

$$BTC/day^* = \theta \cdot (\beta \cdot \rho)/\delta \tag{2}$$

Hayes (2015a,b) sets $\rho = 1000$ GH/s even though the actual hashing power of a miner is likely to deviate greatly from this value. However, Hayes (2015a,b) argues that this level tends to be a good standard of measure.

A lower bound: *the marginal cost of bitcoin production*

The cost of mining per day, E_{day} can be expressed as follows:

$$E_{day} = (\text{price per kWh} \cdot 24\, hr_{day} \cdot \text{W per GH/s})(\rho/1000\, GH/s) \quad (3)$$

Assuming that the bitcoin market is a competitive market, the marginal product of mining should be equal to its marginal cost, so that the $/BTC (equilibrium) price level is given by the ratio of (cost/day) / (BTC/day):

$$p^* = E_{day}/(BTC/day^*) \quad (4)$$

⇒ This price level can be though as a price lower bound, below which a miner would operate at a marginal loss and would probably stop mining.

A lower bound: *the marginal cost of bitcoin production*

Example: use the world average electricity cost ≈ 13.5 cents/KWh, the latest energy efficiency of bitcoin mining hardware ≈ 0.25 J/GH.

⇒ the average cost per day for a 1000 GH/s mining rig is:

$$\begin{aligned} E_{day} &= (\text{price per kWh} \cdot 24\, hr_{day} \cdot \text{W per GH/s})(\rho/1000\, GH/s) \\ &= (0.135 \cdot 24 \cdot 0.25) \cdot (1,000/1,000) = 0.81\$/day \end{aligned}$$

The number of bitcoins that a 1000 GH/s of mining power can find in a day with a current difficulty of 1364422081125 is equal to

$$\begin{aligned} BTC/day^* &= \theta \cdot (\beta \cdot \rho)/\delta = \\ &= 0.0000201165676116943 \cdot (12,5 \cdot 1\mathrm{e}^{12})/1364422081125 \\ &= 0.000184295679925413\, BTC/day. \end{aligned}$$

The \$/BTC price is given by equation (4):

$$\begin{aligned} p^* &= E_{day}/(BTC/day^*) = \\ &= (0.81\$/day)/(000184295679925413 BTC/day) \\ &\approx 4395.11\, \$/BTC, \end{aligned}$$

Modelling bitcoin price dynamics

Most macro-financial analyses devoted to bitcoin prices employ Vecto-AutoRegression (VAR) models,

$$\Delta \mathbf{Y}_{t-1} = \alpha + \Phi_1 \Delta \mathbf{Y}_{t-1} + \Phi_2 \Delta \mathbf{Y}_{t-2} + \ldots + \Phi_p \Delta \mathbf{Y}_{t-p} + \varepsilon_t \quad (5)$$

and Vector Error Correction (VEC) models,

$$\Delta \mathbf{Y}_{t-1} = \alpha + \mathbf{B}\Gamma \mathbf{Y}_{t-1} + \zeta_1 \Delta \mathbf{Y}_{t-1} + \zeta_2 \Delta \mathbf{Y}_{t-2} + \ldots + \zeta_{p-1} \Delta \mathbf{Y}_{t-(p-1)} + \varepsilon_t \quad (6)$$

where **B** are the factor loadings, while Γ the cointegrating vector.

Kristoufek (2013) is the first author to propose a multivariate approach: hefound a significant bidirectional relationship, where Google trends search queries influence prices and viceversa, suggesting that speculation and trend chasing dominate the bitcoin price dynamics.

Modelling bitcoin price dynamics

Glaser et al. (2014) extended previous research by studying the aggregated behavior of new and uninformed Bitcoin users within the time span from 2011 to 2013, to identify why people gather information about Bitcoin and their motivation to subsequently participate in the Bitcoin system.

The main novelty is the use of regressors that are related to both bitcoin **attractiveness** and bitcoin **supply and demand**:

- *daily BTC price data*,
- *daily exchange volumes* in BTC,
- *Bitcoin network volume*, which includes all Bitcoin transfers caused by monetary transactions within the Bitcoin currency network,
- *daily views on the English Bitcoin Wikipedia page* as a proxy for measuring user attention,
- *dummy variables for 24 events gathered from* `https://en.bitcoin.it/wiki/History`.

Modelling bitcoin price dynamics

$\rightarrow$ Glaser et al. (2014) are the first to consider both exchange (EV) and network volumes (NV): their idea is that if a customer want to buy bitcoin to pay for goods or services, exchange and network volumes will share similar dynamics, otherwise only exchange-based volumes will be affected.

$\Rightarrow$ They found that the both increases in Wikipedia searches and in exchange volumes do not impact network volumes, and there is no migration between exchange and network volumes, so that they argued that (uninformed) users mostly stay within exchanges, holding Bitcoin only as an alternative investment and not as a currency.

$\Rightarrow$ Glaser et al. (2014) found that Bitcoin users seem to be positively biased towards Bitcoin, because important negative events, like thefts and hacks, did not lead to significant price corrections.

Modelling bitcoin price dynamics

Bouoiyour and Selmi (2015), Bouoiyour et al. (2015) and Kancs et al. (2015) are the first studies to consider three sets of drivers to model bitcoin price dynamics:

- **technical drivers (bitcoin supply and demand)**,
- **attractiveness indicators**
- and **macroeconomic variables**.

In general, all papers confirm that bitcoin attractiveness factors are still the main drivers of bitcoin price, followed by traditional supply and demand related variables, while global macro-financial variables play no role.

Example: Bouoiyour and Selmi (2015) use these variables: ...

Modelling bitcoin price dynamics

Variable	*Explanation*
	Technical drivers
The exchange-trade ratio (ETR)	Bitcoins are used primarily for two purposes: purchases and exchange rate trading. The Blockchain website provides the total number of transactions and their volume excluding the exchange rate trading. In addition, the ratio between volume of trade (primarily purchases) and exchange transactions is also provided.
Bitcoin monetary velocity (MBV)	It is the frequency at which one unit of bitcoin is used to purchase tradable or non-tradable products for a given period. In the Bitcoin system, the monetary velocity of BitCoin circulation is proxied by the so-called *BitCoin days destroyed*. This variable is calculated by taking the number of BitCoins in transaction and multiplying it by the number of days since those coins were last spent.
The estimated output volume (EOV)	It is similar to the total output volume with the addition of an algorithm which tries to remove change from the total value. This estimate should reflect more accurately the true transaction volume. A negative relationship between the estimated output volume and bitcoin price is expected.
The Hash Rate	The estimated number of giga-hashes per second (billions of hashes per second) the bitcoin network is performing. It is an indicator of the processing power of the Bitcoin network
	Attractiveness indicators
Investors' attractiveness (TTR)	daily Bitcoin views from Google, because it is able to properly depict the speculative character of users
	Macroeconomic variables
The gold price (GP)	Bitcoin does not have an underlying value derived from consumption or production process such as gold.
The Shangai market index (SI)	The Shangai market is considered one of the biggest player in Bitcoin economy and it is considered as a potential source of Bitcoin price volatility.

Modelling bitcoin price dynamics

$\Rightarrow$ Using a dataset spanning between 05/12/2010 and 14/06/2014, Bouoiyour and Selmi (2015) found that in the short-run, the investors attractiveness, the exchange-trade ratio, the estimated output volume and the Shangai index have a positive and significantly impact on Bitcoin price, while the monetary velocity, the hash rate and the gold price have no effect.

$\Rightarrow$ Instead, in the long-run, only the exchange-trade ratio and the hash rate have a significant impact on bitcoin price dynamics.

These results hold also with the inclusion of a dummy variable to account for the bankruptcy of a major Chinese bitcoin trading company in 2013, with oil prices, the Dow Jones index and a dummy variable to consider the closure of the Road Silk by the FBI in October 2013.

Detecting Bubbles and explosive behavior in bitcoin prices

1) Testing for a single bubble: LPPL models. The expected value of the asset log price in a upward trending bubble according to the LPPL equation is given by,

$$E[\ln p(t)] = A + B(t_c - t)^{\beta} + C(t_c - t)^{\beta} \cos[\omega \ln(t_c - t) - \phi] \quad (7)$$

where $A > 0$ is the value of $[\ln p(t_c)]$ at the critical time t_c which is interpreted as the end of the bubble,

$B < 0$ the increase in $[\ln p(t)]$ over the time unit before the crash if C were to be close to 0

$C \neq 0$ is the proportional magnitude of the oscillations around the exponential growth, % and is bounded for the hazard rate to be $+$

$0 < \beta < 1$ to ensure a finite price at the critical time t_c of the bubble and quantifies the power law acceleration of prices,

ω is the frequency of the oscillations during the bubble,

while $0 < \phi < 2\pi$ is a phase parameter.

Detecting Bubbles and explosive behavior in bitcoin prices

Financial bubbles are defined in the LPPL model as transient regimes of faster-than- exponential price growth resulting from positive feedbacks, and these regimes represent "positive bubbles".

Example: Conditions for a (positive) bubble to occur within this framework:

1. $0 < \beta < 1$, which guarantees that the crash hazard rate accelerates.
2. The second major condition is that the crash rate should be non-negative, as highlighted by van Bothmer and Meister (2003),
$$b \equiv -B\beta - |C|\sqrt{\beta^2 + \omega^2} \geq 0.$$
3. Lin et al. (2014) added a third condition, requiring that the residuals from fitting equation (7) should be stationary.

$\Rightarrow$ MacDonell (2014) used the LPPL model to forecast successfully the bitcoin price crash that took place on December 4, 2013

Detecting Bubbles and explosive behavior in bitcoin prices

2) Testing for a multiple bubbles: the Generalized-Supremum ADF test (GSADF).

Tests specifically designed for detecting multiple bubbles were recently proposed by Phillips and Yu (2011), Phillips et al. (2011) and Phillips et al. (2015) and they share the same idea of using sequential tests with rolling estimation windows.

More specifically, *these tests are based on sequential ADF-type regressions* using time windows of different size, and they can consistently identify and date-stamp multiple bubble episodes even in small sample sizes.

We will focus below on the *Generalized-Supremum ADF test (GSADF)* proposed by Phillips, et al. (2015) -PSY henceforward- which builds upon the work by Phillips and Yu (2011) and Phillips et al. (2011), because it has better statistical properties in detecting multiple bubble than the latter two tests.

Detecting Bubbles and explosive behavior in bitcoin prices

This test employs an ADF regression with a rolling sample, where the starting point is given by the fraction r_1 of the total number of observations, the ending point by the fraction r_2, while the window size by $r_w = r_2 - r_1$. The ADF regression is given by

$$y_t = \mu + \rho y_{t-1} + \sum_{i=1}^{p} \phi_{r_w}^{i} \Delta y_{t-i} + \varepsilon_t \tag{8}$$

where the null hypothesis is of a unit root $\rho = 1$ versus an alternative of a mildly explosive autoregressive coefficient $\rho > 1$.

The backward sup ADF test proposed by PSY (2015) fixes the endpoint at r_2 while the window size is expanded from an initial fraction r_0 to r_2, so that the test statistic is given by:

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2} \tag{9}$$

Detecting Bubbles and explosive behavior in bitcoin prices

The generalized sup ADF (GSADF) test is computed by repeatedly performing the BSADF test for each $r_2 \in [r_0, 1]$:

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1]} BSADF_{r_2}(r_0) \qquad (10)$$

PSY (2015, Theorem 1) provides the limiting distribution of (10) under the null of a random walk with asymptotically negligible drift (vs an alternative of a mildly explosive process), while critical values are obtained by numerical simulation.

If the null hypothesis of no bubbles is rejected, it is then possible to date-stamp the starting and ending points of one (or more) bubble(s) in a second step...

Detecting Bubbles and explosive behavior in bitcoin prices

More specifically,

$\rightarrow$ the *starting point* is given by the date -denoted as T_{r_e}- when the sequence of BSADF test statistics crosses the critical value from below,

$\rightarrow$ whereas the *ending point* -denoted as T_{r_f}- when the BSADF sequence crosses the corresponding critical value from above:

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \left\{ r_2 : BSADF_{r_2}(r_0) > cv_{r_2}^{\beta_T} \right\} \tag{11}$$

$$\hat{r}_f = \inf_{r_2 \in [\hat{r}_e + \delta \log(T)/T, 1]} \left\{ r_2 : BSADF_{r_2}(r_0) < cv_{r_2}^{\beta_T} \right\} \tag{12}$$

where $cv_{r_2}^{\beta_T}$ is the $100(1-\beta_T)\%$ right-sided critical value of the BSADF statistic based on $\lfloor Tr_2 \rfloor$ observations, $\lfloor \cdot \rfloor$ is the integer fun.

δ is a tuning parameter which determines the minimum duration for a bubble and is usually set to 1, see PSY (2015) and references therein, thus implying a minimum bubble-duration condition of $\ln(T)$ observations.

Detecting Bubbles and explosive behavior in bitcoin prices

Malhotra and Maloo (2014) tested for the presence of multiple bubbles using the GSADF test with data ranging from mid-2011 till February 2014:

⇒ they found evidence of explosive behaviour in the bitcoin-USD exchange rates during ***August – October 2012*** and ***November, 2013 – February, 2014***.

⇒ They suggested that the first episode of bubble behavior (August – October 2012) could be attributed to the sudden increase in media attention towards bitcoin,

⇒ whereas the second episode to a large set of reasons including the US debt ceiling crisis, the shutdown of Silk Road by the FBI, the rise of Chinese exchange BTC-China, and the increasing number of warnings issued by regulatory authorities and central banks worldwide following the shutdown of the Japanese exchange Mt.Gox.

Price Discovery

Brandvold et al. (2015) are the first (and so far the only ones) to study the price discovery process in the Bitcoin market, which consists of several independent exchanges.

This topic is frequently discussed in the bitcoin community because knowing which exchange reacts most quickly to new information (thus reflecting the value of Bitcoin most precisely), is clearly of outmost importance for both short-term traders and long-term investors.

Brandvold et al. (2015) used the method by de Jong et al. (2001) becauseit has the advantage that the information share is uniquely defined, unlike Hasbrouck's (1995) model, and it takes the variance of price shocks into account, unlike Gonzalo and Granger (1995), so that a price series with low innovation variance gets a low information share.

Price Discovery

It is possible to show that the information share for exchange i (= how much information is generated by the price change of exchange i in %) is given by:

$$IS_i = \frac{(\sigma^2 + \psi_i)\pi_i}{\sigma^2} = \pi_i \left(1 + \frac{\psi_i}{\sigma^2}\right) \quad \text{where} \tag{13}$$

- σ^2 can be computed as the variance of the aggregated return of the Bitcoin exchanges,
- i refers to exchange i for $i = 1, ..., n$,
- π_i is the *activity share of an exchange*, defined as the *fraction of trades that happened on exchange i*, or simply, *the probability that a trade took place on exchange i*,
- ψ_i is the is the covariance between the fundamental news component and the exchange idiosyncratic component, and is computed by numerical optimization, see Brandvold et al. (2015) for details.

Price Discovery

Brandvold et al. (2015) used data from seven exchanges: Bitfinex, Bitstamp, BTC-e (Btce), BTC China(Btcn) and Mt.Gox (Mtgox), Bitcurex and Canadian Virtual Exchange (Virtex). Data covered the period April 1st 2013–February 25th 2014, till bankruptcy of Mtgox.

They found that the two exchanges with positive ψ for the entire period were Btce and Mtgox, thus indicating that these exchanges were more informative than their competitors.

Similar evidence was provided by the information share, which was highest for Btce and Mtgox (0,322 and 0.366, respectively).

$\Rightarrow$ Information shares change over time: for example, the information share of Btcn first increased from 0.040 in April 2013 to 0.325 in December 2013 because some large Chinese companies (like Baidu) started accepting Bitcoin as payment, but then its information share fell to 0.124 in January 2014 after the Chinese government banned payment companies from clearing Bitcoin.

Conclusions

Financial modelling of cryptocurrencies has only started and there are several possible avenues for further research:

- Econometric methods for market and credit risk management with cryptocurrencies prices are almost non-existent.
- Despite the changes in local regulations, arrival of new investors, police intervention (Silk Road, BTC-e) and massive improvements in mining hardware, there is no research work dealing with structural breaks and long memory.
- All models examined so far are (log-)linear but, considering the behavior of bitcoin prices, nonlinear models could be useful.
- Multi-disciplinary analyses are needed: *IT related papers* focused mainly on electricity costs and energy and computational efficiency, whereas *economic related papers* rarely considered these factors.

Conclusions

More details can be found here:

1. Fantazzini, D., Nigmatullin, E., Sukhanovskaya, V., & Ivliev, S. (2016). Everything you always wanted to know about bitcoin modelling but were afraid to ask. Part 1. *Applied Econometrics*, 44, 5-24. Available at: `https://ideas.repec.org/a/ris/apltrx/0301.html`

2. Fantazzini, D., Nigmatullin, E., Sukhanovskaya, V., & Ivliev, S. (2017). Everything you always wanted to know about bitcoin modelling but were afraid to ask. Part 2. *Applied Econometrics*, 45, 5-28. Available at: `https://ideas.repec.org/a/ris/apltrx/0308.html`

3. I am writing a textbook about it . . . stay tuned . . .